The data obtained on the radiation spectrum fields can be utilized to compute the integrated optical characteristics of a material and the integrated radiation field distribution in a layer that are needed to determine the heat source for an analytic description of the temperature fields under IR-expose.

NOTATION

R'_λ, T'_λ $R'_{\lambda\infty}$ are directional-hemispheric $(\theta, \varphi, 2\pi)$ thermoradiational characteristics; R_λ, T_λ, $R_{\lambda\infty}$ are bihemispherical (2π; 2π) thermoradiational characteristics; L_λ, $\bar{k}_\lambda$, s_λ, ε_λ are optical characteristics-the effective attenuation, absorption "back-scattering", and extinction coefficients, m^{-1}; C_1 and C_2 are Duntley parameters, E_λ is the density of diffuse, and E'_λ the density of directional monochromatic radiation flux incident at a certain angle θ, W/m^2; w'_λ and w_λ are the magnitudes of the absorbed radiation energy flux under directional and diffuse exposure, W/m^3; $E_\lambda{}^0$ is the magnitude of the spatial irradiance, W/m^2; and q_λ is the density of the resultant flux, W/m^2.

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HEAT TRANSFER IN ANISOTROPIC METALLIC MEDIA

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The temperature distribution over the cross-section of a metallic sample with anisotropic kinetic coefficients heated by an electric current in liquid helium has been studied.

As is known, low-temperature phenomena of heat and charge transfer in metals are determined by the degree of nonequilibrium of the conduction electrons, their dispersion law, which results in the interaction of these phenomena, and their distinct influence on each other [1-5]. This appears to be especially strong under conditions of anisotropy, both natural, crystalline anisotropy, and also artificial anisotropy introduced, for example, by a strong external magnetic field. Thus, in metals belonging to a cubic crystallographic system, kinetic coefficients that are described by scalar values in the absence of a magnetic field become tensors of the second rank in a strong magnetic field; the latter results in the emergence of many cross effects in thermal electrotransfer.

In the present work results are given of the study of a steady-state problem of heat transfer and of the effect of the conditions for the Joule power dissipated in the volume of a sample on the temperature distribution over the cross-section of a metallic single-crystal sample in a strong transverse magnetic field with temperature-dependent kinetic coefficients of the metal.

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As an object of study we have chosen a sample having a square cross-section with a current flowing along the sample parallel to the OX-axis with density j_x that is constant over the cross-section, i.e., we use an approximation of a specified current that is typical for low-temperature transfer phenomena, with the current along the other directions being absent in the entire volume of the sample, i.e., $j_y = j_z = 0$. The sample is immersed in liquid helium so that one pair of faces [normal with respect to the vector of the magnetic field intensity $H = H_z$ ($H_x = H_y = 0$)] is adiabatically isolated from the environment, and heat flow along the OZ-axis is absent in the entire volume of the sample ($q_z = 0$). The other pair of lateral faces (normal with respect to the OZ-axis, with coordinates $y = \pm d/2$, where d is the the transverse dimension of the sample) is open and has the temperature of liquid helium, equal to 4.2 K. The heat is dissipated through these faces. In this case, when the released joule power is dissipated in a thermostat, the temperature distribution over the cross-section of the sample is determined from the steady-state equation of heat transfer, which in differential form is

$$\operatorname{div} q - jE = 0, \tag{1}$$

where q is the heat flow through the lateral surface in the direction of external normal toward this surface, and jE is the scalar product of the vectors of the current density and the electric field intensity, representing a heat source in the given volume. To formulate the problem in closed form, Eq. (1) should be supplemented by the generalized equations of charge and heat transfer, which in differential form are written as

$$E_i = \rho_{ik} j_k + \alpha_{ik} \frac{\partial T}{\partial X_k}, \tag{2}$$

$$q_i = \pi_{ik} j_k - \varkappa_{ik} \frac{\partial T}{\partial X_k}, \tag{3}$$

where i, k = x, y, z, and summation is assumed over repeated indices; E_i and q_i are the components of the vectors of the electric field intensity and heat flow along the i-th direction of the Cartesian system of coordinates; ρ_{ik}, α_{ik}, and $\varkappa_{ik}$ are components of the electrical resistance tensors, thermo electromotive force, and heat conduction; π_{ik} is the component of the tensor describing the Peltier effect; and $\partial T/\partial X_k$ temperature gradient along one of the directins (x, y, z).

By using the condition that the dimensions of the sample along the direction of the electric field are much larger than the transverse dimensions, we apply the approximation of an infinitely long sample, for which the distribution over T or the temperature gradient along the long axis is absent, i.e., $\partial T/\partial x = 0$. At first, as a sample material we consider a simple noncompensated metal with a closed Fermi surface in which elastic interaction with impurities dominates in the scattering of conduction electrons; this allow us to introduce a temperature-independent electron relaxation time τ, i.e., we can use the so-called τ-approximation for describing the kinetic coefficients of a metal. By eliminating from the expression for the flow q_y the gradient $\partial T/\partial z$, we obtain from (1) the following equation, by solving which we determine the temperature distribution along the OY-axis:

$$\left(\pi_{yx} - \pi_{zx} \frac{\varkappa_{yz}}{\varkappa_{zz}}\right)'_T j_x \frac{\partial T}{\partial y} - \left(\varkappa_{yy} - \varkappa_{yz} \frac{\varkappa_{zy}}{\varkappa_{zz}}\right)'_T \left(\frac{\partial T}{\partial y}\right)^2 - \left(\varkappa_{yy} - \varkappa_{yz} \frac{\varkappa_{zy}}{\varkappa_{zz}}\right) \times$$
$$\times \frac{\partial^2 T}{\partial y^2} = \rho_{xx} j_x^2 + \alpha_{xy} j_x \frac{\partial T}{\partial y} + \alpha_{xz} \frac{\pi_{zx}}{\varkappa_{zz}} j_x^2 - \alpha_{xz} \frac{\varkappa_{zy}}{\varkappa_{zz}} \frac{\partial T}{\partial y} j_x. \tag{4}$$

We introduce the notation

$$D = \alpha_{xy} - \alpha_{xz} \frac{\varkappa_{zy}}{\varkappa_{zz}} - \left(\pi_{yx} - \pi_{zx} \frac{\varkappa_{yz}}{\varkappa_{zz}}\right)'_T; \; F = \varkappa_{yy} - \varkappa_{yz} \frac{\varkappa_{zy}}{\varkappa_{zz}},$$

where the prime and T next to the parentheses denote the differentiation of the expression inside the parentheses with respect to temperature. The differential equation obtained contains coefficients for $\partial T/\partial y$, depending on temperature. However, taking account of the fact that in the τ-approximation the kinetic coefficients of the metal in (4) have a simple, i.e., explicit linear or quadratic dependence on T $(\alpha_{ik} \infty T;\ \varkappa_{ik} \infty T;\ \pi_{ik} \infty \alpha_{ik} T \infty T^2)$, it is easy to reduce Eq. (4) to a form in which the coefficients for the derivatives are constants. Indeed, since all the coefficients for $\partial T/\partial y$ in (4) contain the temperature only in the first power, it can be readily factored out, and after regrouping terms, Eq. (4) can be represented in the form

$$\left(\pi_{yx} - \pi_{zx} \frac{\varkappa_{yz}}{\varkappa_{zz}}\right)''_{TT} j_x \frac{\partial}{\partial y}\left(\frac{T^2}{2}\right) - F'_T \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\frac{T^2}{2}\right) - (\alpha_{xy})'_T\, j_x T \frac{\partial T}{\partial y} +$$
$$+\left(\alpha_{xz} \frac{\varkappa_{zy}}{\varkappa_{zz}}\right)'_T j_x T \frac{\partial T}{\partial y} = \rho_{xx} j_x^2 + \alpha_{xz} \frac{\pi_{zx}}{\varkappa_{zz}} j_x^2,$$

or after regrouping,

$$-F'_T \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{T^2}{2}\right)\right) + D'_T \frac{\partial}{\partial y}\left(\frac{T^2}{2}\right) - \left(\rho_{xx} + \alpha_{xz} \frac{\pi_{zx}}{\varkappa_{zz}}\right) j_x^2 = 0. \tag{5}$$

The equation obtained is an equation of first order with respect to the quantity $\frac{\partial}{\partial y}\left(\frac{T^2}{2}\right)$. It should be noted that in the expression describing the source of heat, ρ_{xx} does not depend on temperature, since a slight warming-up of the sample does not violate the elastic character of the electron scattering, and ρ_{xx} agrees with the residual resistance ρ_0 in order of magnitude, while $\frac{\alpha_{xz}}{\varkappa_{zz}} \pi_{zx}$, in turn, is a quadratic function of the temperature.

We estimate the values of the kinetic coefficients of the metal entering into (5). Thus, for the residual resistance $\rho_0 \sim 10^{-16}$ ohm·cm, from the Wiedemann-Franz law and the Lorentz number, we obtain $\varkappa \sim 2.4 \cdot 10^2$ TW/(cm·K²). The order ofmagnitude of α_{xz} for the given case is 10^{-8} V/K. In order to estimate the quantity π_{zx} we use the condition of the symmetry of the kinetic coefficients in a strong magnetic field [4]:

$$\pi_{ik} = \xi_{il} \rho_{lk};\quad \xi_{il}(H) = -T\beta_{il}(H), \tag{6}$$

where β_{il} is a proportionality constant between the current density and the temperature gradient in (2) rewritten in the form

$$j_i = \sigma_{ik} E_k + \beta_{il} \frac{\partial T}{\partial X_l}.$$

By using expressions for the tensors β and ρ in the presence of a strong magnetic field [4]:

$$\beta = \begin{pmatrix} \gamma^2 \cdot C_{xx} & \gamma \cdot C_{xy} & \gamma \cdot C_{xz} \\ -\gamma \cdot C_{xy} & \gamma^2 \cdot C_{yy} & \gamma \cdot C_{yz} \\ -\gamma \cdot C_{xz} & -\gamma \cdot C_{yz} & C_{zz} \end{pmatrix},\quad \rho = \begin{pmatrix} B_{xx} & \gamma^{-1} \cdot B_{xy} & B_{xz} \\ \gamma^{-1} \cdot B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{pmatrix},$$

where $\gamma = (\omega\tau)^{-1}$, ω is the cyclotron frequency, τ is the relaxation time of the charge carriers; C and B are the values of the corresponding kinetic coefficients for H = 0, it is easy to show that π_{zx} for the given case agrees in order of magnitude with π in the absence of a magnetic field, i.e., $\pi_{zx} \simeq \pi \sim \alpha_T T^2 \sim 16 \cdot 10^{-8}$ V for T = 4.2 K. Finally, by comparing the numerical values of the quantities $\varkappa_{xz}\varkappa_{zz}^{-1}\pi_{zx}$ and ρ_{xx}, it is easy to notice that ρ_{xx} is larger by seven orders of magnitude and without a noticable loss of accuracy of the solution we can neglect the term containing $\alpha_{xz}\varkappa_{zz}^{-1}\pi_{zx}$, in the expression for the heat source. The equation obtained is easily solved in order to determine the second constant of integration with the symmetry condition $\partial T/\partial y = 0$ for $y = 0$ being used. As a result we obtain the following temperature distribution along the OY-axis:

$$T = \sqrt{2}\left\{\frac{\rho_{xx}j_x^2}{D'_T j_x}\left[-\frac{F'_T}{D'_T j_x}\exp\left(-\frac{D'_T j_x}{F'_T}y\right)-y\right]+\frac{T_0^2}{2}-\frac{\rho_{xx}j_x^2}{D'_T j_x}\left[-\frac{F'_T}{D'_T j_x}\exp\left(-\frac{D'_T j_x}{F'_T}\frac{d}{2}\right)-\frac{d}{2}\right]\right\}^{\frac{1}{2}}. \tag{7}$$

Consequently, the temperature gradient at any point is described by the expression

$$\frac{dT}{dy}=\frac{\sqrt{2}}{2}\left\{\frac{\rho_{xx}j_x^2}{D'_T j_x}\left[-\frac{F'_T}{D'_T j_x}\exp\left(-\frac{D'_T j_x}{F'_T}y\right)-y\right]+\frac{T_0^2}{2}-\frac{\rho_{xx}j_x^2}{D'_T j_x}\left[-\frac{F'_T}{D'_T j_x}\exp\left(-\frac{D'_T j_x}{F'_T}\frac{d}{2}\right)-\frac{d}{2}\right]\right\}^{-\frac{1}{2}}\times\left[\frac{\rho_{xx}j_x^2}{D'_T j_x}\exp\left(-\frac{D'_T j_x}{F'_T}y\right)-1\right]. \tag{8}$$

We simplify the given expression and estimate the values entering into it. Thus, for the closed Fermi surface from expressions for ρ and β we obtain the order of value π_{yx}: $\pi_{yx}=\gamma\pi$, while and $\varkappa_{yz}$ are determined by expressions $\alpha_{xy}=\gamma\alpha$; $\varkappa_{yy}=\gamma^2\varkappa$; $\varkappa_{yz}=\gamma\varkappa$, with $\alpha \approx 10^{-8}$ V/K, and $\varkappa \simeq 9.8\cdot10^2$ W/cm·K). We estimate the value of the expression under the exponential sign

$$\frac{D'_T j_x}{F'_T}=\frac{\left[\alpha_{xy}-\alpha_{xz}\frac{\varkappa_{zy}}{\varkappa_{zz}}-(\pi_{yx}-\pi_{xz}\varkappa_{yz}\varkappa_{zz}^{-1})'_T\right]'_T j_x}{(\varkappa_{yy}-\varkappa_{yz}\varkappa_{zy}\varkappa_{zz}^{-1})'_T}. \tag{9}$$

In the magnetic field H = 100 kOe, $\delta\tau = 7.2\cdot10^2$, and the argument of the exponent is equal to $3.6\cdot10^{-4}$ cm^{-1}, which allows us to expand the exponent on a characteristic length equal to the transverse dimension of the sample in a quickly converging series and to restrict ourselves to the first nonvanishing terms. As a result, the expression for $\partial T/\partial y$ for $y = d/2$ assumes the form

$$\frac{dT}{dy}=\frac{\rho_{xx}j_x^2}{(\varkappa_{yy}-\varkappa_{yz}\varkappa_{zy}\varkappa_{zz}^{-1})'_T T_0}\,\frac{d}{2}. \tag{10}$$

The same consideration is applicable for metals with a more complicated law of dispersion of conduction electrons, for example, when the Fermi surface is not closed (a corrugated cylinder open along the transport direction OX). However, in order to convert from Eq. (4) to an equation with constant coefficients it is necessary to compare ρ_{xx} and $\alpha_{xz}\varkappa_{zz}^{-1}\pi_{zx}$ in magnitude under the new conditions of openness of the Fermi surface. For the case of a metal with Fermi surface open along the OX-axis, tensors β and ρ are of the form

$$\beta=\begin{pmatrix}\gamma^2C_{xx} & \gamma C_{xy} & \gamma C_{xz}\\ \gamma C_{yx} & C_{yy} & C_{yz}\\ \gamma C_{zx} & C_{zy} & C_{zz}\end{pmatrix};\quad \rho=\begin{pmatrix}\gamma^{-2}B_{xx} & \gamma^{-1}B_{xy} & \gamma^{-1}B_{xz}\\ \gamma^{-1}B_{yx} & B_{yy} & B_{yz}\\ \gamma^{-1}B_{zx} & B_{zy} & B_{zz}\end{pmatrix}. \tag{11}$$

By using (6) it is easy to find out that $\pi_{zx}\simeq\gamma^{-1}\pi$. With account for the fact that $\rho_{xx}\propto(\omega\tau)^2\rho_0$, $\alpha_{xz}\simeq\omega\tau\alpha$ and $\varkappa_{zz}\simeq\varkappa$, we obtain that the relationship between the values ρ_{xx} and

$\alpha_{xz}\pi_{zx}\varkappa_{zz}^{-1}$ for the case of an open Fermi surface does not change since both values increase by the factor $(\omega\tau)^2$. This allows us, with the previous accuracy, to ignore the term $\alpha_{xz}\pi_{zx}\varkappa_{zz}^{-1}$ as compared with ρ_{xx} (4).

In order to estimate the temperature gradient on the lateral face of a real metal sample with open Fermi surface we should simplify Eq. (8). In order to do this we list the values of parameters entering into the argument of the exponent in (8), determining beforehand from (11) and (6) the values of the components of the Peltier tensor π_{xz} and π_{yx}, which in order of magnitude constitute $\pi_{xz}\simeq\gamma\pi$, $\pi_{yx}\simeq\pi\gamma^{-1}$ and $\alpha_{xy}\simeq\alpha\gamma^{-1}$; $\alpha_{xz}\simeq\alpha\gamma^{-1}$; $\varkappa_{yy}\simeq\varkappa_{xz}\simeq\varkappa_{yz}\simeq\varkappa$. Substituting the obtained values of the components of tensors of kinetic coefficients of the metal with the open Fermi surface in (9) and retaining the lowest powers with respect to the parameter γ, we obtain that the argument of the exponent remains a quantity of the order of 10^{-4} cm^{-1} as before. Therefore, even in this case we can simplify (8) by expanding the exponent into a series; after that, the expression for the temperature gradient on the lateral surface of the sample assumes form (10). If it is taken into account that the functional dependence of the kinetic coefficients on the magnetic field (in metals with a closed Fermi surface, ρ_{xx} does not depend on H, but $\varkappa_{yy}\propto H^{-2}$ while in metals with the Fermi surface open along the OX-axis $\varkappa_{yy}\sim\varkappa$ but $\rho_{xx}\propto H^2$), then it becomes apparent that the temperature gradient on the lateral surface of the sample both for the case of closed and open electron orbits is the same in order of magnitude. The estimated transverse temperature gradient in the anisotropic medium causes a thermoelectric contribution of the transport current to the electric field according to Eq. (2). In this case the sign and magnitude of this contribution are determined by the direction of the magnetic field and by peculiarities of the band structure of the system of charge carriers. Therefore, in a metal with open Fermi surface, this contribution is much larger than for a metal with a closed Fermi surface because the components of the tensor of the thermoelectric force α_{xy}^{op} and α_{xy}^{cl} differ by a factor of $(\omega\tau)^2$ i.e., $\alpha_{xy}^{op} \simeq \gamma^{-1}\alpha$, $\alpha_{xy}^{cl} \simeq \gamma\alpha$. This, however, does not mean that in the given approximation the fraction of the thermoelectric effect in the overall potential difference being measured in the case of the open Fermi surface is larger than for the closed, since the ohmic voltage drop also increases in the given case by a factor of $(\omega\tau)^2$ due to the diagonal component of the tensor of electric resistance.

Therefore, the conducted analysis shows that in the course of the low-temperature experiment on galvanomagnetic properties of metallic media, in addition to the harmful effect of the intrinsic field of the transport current on the charge transfer, it is necessary to conduct studies on the magnitude and effect of thermoelectric phenomena on the desired ohmic voltage drop, which is particularly timely for low-ohmic high-purity metals, requiring large densities of the transport current.

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